INVESTIGATION OF THE SPECTRUM OF THE GENERALIZED FRIDRIXS MODEL OPERATOR

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Abstract: This article examines the spectral properties of the generalized Fridrixs model operator, with particular emphasis on the essential spectrum, discrete eigenvalues, and the dependence of spectral characteristics on perturbation parameters. The study provides an analytical review of existing research on generalized Fridrixs models and summarizes the main approaches used to investigate the existence and number of eigenvalues. Particular attention is paid to finite-rank perturbations and their role in the formation of the discrete spectrum. The results of recent studies are analyzed to identify the main trends in the spectral theory of generalized Fridrixs models.
Keywords: generalized Fridrixs model, spectral theory, operator spectrum, essential spectrum, eigenvalue, finite-rank perturbation

Introduction

In modern mathematical physics and operator theory, the investigation of the spectral properties of self-adjoint operators is one of the important areas of research. In particular, the Fridrixs model and its generalizations are of significant importance for the mathematical description of interacting systems in quantum mechanics, the study of the existence and location of eigenvalues, and the analysis of the relationship between continuous and discrete spectra.

The generalized Fridrixs model is a more sophisticated form of the classical Fridrixs model in which finite-rank perturbations are introduced into the main operator. Such models make it possible to investigate the essential spectrum of an operator, discrete eigenvalues located outside the essential spectrum, and the dependence of spectral properties on model parameters.

In recent years, research on generalized Fridrixs models has focused mainly on the existence of eigenvalues, their number, their location outside the essential spectrum, and the influence of parameters on the spectral structure.

Literature Review

The Fridrixs model was originally developed for the mathematical description of interacting systems in quantum mechanics. Subsequently, its generalized forms became important research objects in operator theory.

Studies by Abdullaev, Ikromov, and Lakaev investigated embedded eigenvalues and resonances in generalized Fridrixs models. These studies provided a theoretical basis for a deeper analysis of the spectral structure of such models.

Akchurin investigated the spectral properties of generalized Fridrixs models, with particular attention to the influence of perturbations on the spectrum and the location of eigenvalues.

Muminov developed approaches for determining the number of eigenvalues of Fridrixs models. His studies also considered lacunas in the essential spectrum and the possibility of eigenvalues occurring in such regions.

In their 2024 study, Muminov and Shadiev investigated conditions for the existence of an eigenvalue in a generalized Fridrixs model. Their work analyzed the spectral properties of the model and the conditions under which eigenvalues may occur.

In a 2025 study, Lakaev, Kurbanov, and Alladustov investigated the number of eigenvalues of a generalized Fridrixs model with a compactly supported perturbation. The study considered the division of the parameter space into separate regions and the constancy of the number of eigenvalues in different parameter regions

Research Methodology

The study employs fundamental methods of operator theory, functional analysis, and spectral analysis. Particular attention is paid to distinguishing between the essential and discrete spectra of the generalized Fridrixs model, determining the influence of perturbations on the spectral structure, and analyzing the existence and number of eigenvalues. The theoretical basis of the study consists of existing research on generalized Fridrixs models. In particular, the fundamental results of spectral theory for finite-rank perturbations and approaches related to spectral analysis are considered.

The generalized Fridrixs model can be considered as a family of operators consisting of a main operator and a perturbation. The principal form of the model considered in the relevant studies is expressed as follows:

$H\mu 1\mu 2(p) = H0(p) + V\mu 1\mu 2$

$\mu 1, \mu 2 \in R.$

That is:

H μιμ2 (p) – generalized Fridrixs model operator;

Ho(p) – the main (free) operator of the model;

Vμιμη - perturbation operator;

μ1, μ2 – real parameters defining the perturbation.

This expression demonstrates that the perturbation is determined by two parameters. This property makes it possible to investigate the spectral characteristics of the model for different parameter values. One of the important characteristics of the generalized Fridrixs model is associated with the finite-rank nature of the perturbation. Such perturbations can preserve the principal spectral characteristics associated with the essential spectrum of an operator. Therefore, the spectral properties of the unperturbed operator are important for investigating the essential spectrum of the model.

In the literature, the essential spectrum of a generalized Fridrixs model is generally described in relation to the spectrum of the principal operator. The perturbation, in turn, may lead to the appearance of discrete eigenvalues outside the essential spectrum. Therefore, distinguishing between the essential and discrete spectra is one of the fundamental stages in the spectral analysis of generalized Fridrixs models.

One of the principal problems in the investigation of generalized Fridrixs models is determining the existence of eigenvalues outside the essential spectrum.bs. yeah. As a result of the perturbation, new eigenvalues that are not present in the spectrum of the principal operator may appear. Such eigenvalues generally constitute the discrete spectrum.

In generalized Fridrixs models, the perturbation parameters have a direct influence on the spectral properties. Different parameter values may lead to different spectral structures. Such an approach has important methodological significance for the systematic investigation of the spectral properties of generalized Fridrixs models.

Results and Discussion

The analysis of the selected scientific literature demonstrates that the spectrum of a generalized Fridrixs model operator involves several interconnected problems. These include the structure of the essential spectrum, the existence of discrete eigenvalues outside the essential spectrum, their number, and their dependence on perturbation parameters. The finite-rank perturbation in the model represents an important property for conducting spectral analysis. At the same time, changes in the parameters

may lead to the appearance or disappearance of eigenvalues. In particular, determining the constancy of the number of eigenvalues by dividing the parameter space into separate regions represents an important result in contemporary research on generalized Fridrixs models.

The scientific significance of investigating the spectrum of generalized Fridrixs models is determined by their connection with numerous problems in quantum mechanics, mathematical physics, and functional analysis. Research based on this model provides a theoretical basis for describing the essential spectrum of an operator, determining the existence of discrete eigenvalues, estimating their number, and identifying the influence of perturbation parameters on the spectrum.Furthermore, investigations of generalized Fridrixs models can be applied to the mathematical modeling of spectral properties of quantum systems

Conclusion

The investigation of the spectrum of generalized Fridrixs model operators represents an important area of modern spectral theory. The analysis of the scientific literature demonstrates that significant results have been obtained concerning the essential spectrum, discrete eigenvalues, resonances, and parametric properties of these models. Previous studies show that finite-rank perturbations play an important role in investigating the spectral structure of generalized Fridrixs models. In particular, determining the existence and number of eigenvalues depending on perturbation parameters constitutes one of the central problems in this field. Future research may focus on a deeper investigation of spectral changes near critical parameter values, the analysis of threshold phenomena, and the behavior of eigenvalues with respect to model parameters.

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