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LINEAR TRANSFORMATIONS IN MINKOWSKI SPACE AND THEIR PHYSICAL INTERPRETATIONS IN SPECIAL RELATIVITY

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ANNOTATSIYA

Ushbu tezis Minkovskiy fazosidagi chiziqli almashtirishlarning maxsus nisbiylik nazariyasidagi matematik va fizik mazmunini o‘rganishga bag‘ishlangan. Minkovskiy fazosining intervali, Lorens almashtirishlari va ularning metrikani saqlash xususiyati tahlil qilinadi. Shuningdek, chiziqli almashtirishlarning turli inersial sanoq sistemalaridagi koordinatalar, vaqt va fazoviy masofalarning o‘zaro bog‘liqligini ifodalashdagi roli ko‘rsatiladi.

Kalit so‘zlar: Minkovskiy fazosi, Lorens almashtirishi, maxsus nisbiylik, chiziqli almashtirish, invariant interval, fazo-vaqt, Lorens guruhi.

АННОТАЦИЯ

В данной работе рассматриваются математические и физические интерпретации линейных преобразований в пространстве Минковского в рамках специальной теории относительности. Анализируются интервалы Минковского, преобразования Лоренца и их свойство сохранять метрику пространства-времени. Показана роль линейных преобразований в описании взаимосвязи пространственных и временных координат в различных инерциальных системах отсчёта.

Ключевые слова: пространство Минковского, преобразование Лоренца, специальная теория относительности, линейное преобразование, инвариантный интервал, пространство-время, группа Лоренца.

ABSTRACT

This thesis examines the mathematical structure and physical interpretation of linear transformations in Minkowski space within special relativity. The Minkowski spacetime interval, Lorentz transformations, and their metric-preserving properties are analyzed. Particular attention is given to the role of linear transformations in relating space and time coordinates between different inertial reference frames.

Keywords: Minkowski space, Lorentz transformation, special relativity, linear transformation, invariant interval, spacetime, Lorentz group.

INTRODUCTION

Special relativity describes physical phenomena in inertial reference frames through the geometry of four-dimensional spacetime. The mathematical formulation introduced by Minkowski unifies three spatial coordinates and time into a single spacetime structure. In this formulation, events are represented by four-vectors, while transformations between inertial frames are described by Lorentz transformations [1, pp. 48–55].

For an event with coordinates

$$x^{\mu}=(c,t,x,yz),$$

the spacetime interval is defined as

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$$s^2=c^2t^2-x^2-y^2-z^2.$$

A fundamental requirement of special relativity is that this interval remains invariant under transformations between inertial frames [2, pp. 21–27]. Therefore, the study of linear transformations in Minkowski space provides a geometric interpretation of the basic principles of special relativity.

METHODS

The mathematical analysis is based on four-dimensional vector spaces, bilinear forms, matrix representations, and invariant quantities. A linear transformation can be written as

$$x'^{\mu}=\Lambda^{\mu}_{\nu}x^{\nu},$$

where $\Lambda$is a 4×4transformation matrix.

For the Minkowski metric

$$\eta=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix},$$

the Lorentz transformation must satisfy

$$\boxed{\Lambda^T\eta\Lambda=\eta}.$$

This condition guarantees that the spacetime interval is invariant:

$$x'^T\eta x'=x^T\eta x.$$

Thus,

$$s'^2=s^2.$$

The preservation of this quadratic form distinguishes Lorentz transformations from ordinary Euclidean rotations [1, pp. 53–60; 3].

RESULTS

Consider two inertial reference frames Sand S', where S'moves with velocity valong the x-axis relative to S. The corresponding Lorentz transformation is

$$\begin{aligned}ct'&=\gamma(ct-\beta x),\\x'&=\gamma(x-\beta ct),\\y'&=y,\\z'&=z,\end{aligned}$$

where

$$\beta=\frac{v}{c},\ \gamma=\frac{1}{\sqrt{1-\beta^2}}.$$

In matrix form,

$$\begin{pmatrix}ct'\\x'\\y'\\z'\end{pmatrix}=\begin{pmatrix}\gamma&-\gamma\beta&0&0\\-\gamma\beta&\gamma&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}ct\\x\\y\\z\end{pmatrix}.$$

The transformation demonstrates that time and space coordinates are not independent quantities. A change of inertial reference frame mixes the temporal coordinate ctwith the spatial coordinate x. This mathematical structure leads directly to the physical effects of time dilation and length contraction [2, pp. 28–35].

The Lorentz transformation also preserves the causal structure of spacetime. Depending on the sign and magnitude of the invariant interval, two events may be timelike, lightlike, or spacelike separated:

$$s^2>0,\ s^2=0,\ s^2<0.$$

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These three cases determine whether a causal signal travelling at or below the speed of light can connect the events [3].

Another important consequence is that physical quantities such as energy and momentum form a four-vector:

$$p^{\mu}=\left(\frac{E}{c},p_x,p_y,p_z\right).$$

Its invariant norm gives

$$p^{\mu}p_{\mu}=\frac{E^2}{c^2}-p^2=m^2c^2,$$

or equivalently,

$$\boxed{E^2=p^2c^2+m^2c^4}.$$

Thus, the same geometric principle that preserves the spacetime interval also produces the relativistic energy–momentum relation [1, pp. 70–75].

DISCUSSION

The results show that Lorentz transformations should not be interpreted merely as algebraic formulas for converting coordinates. They represent symmetry transformations of Minkowski spacetime. Their defining property,

$$\Lambda^T\eta\Lambda=\eta,$$

expresses the invariance of the spacetime geometry under changes between inertial observers.

From the physical perspective, this invariance explains why different inertial observers may assign different values to time and spatial coordinates while agreeing on the invariant spacetime interval. Consequently, time dilation, length contraction, relativity of simultaneity, and relativistic energy–momentum relations can all be understood as consequences of the same underlying spacetime geometry [2, pp. 36–43; 4].

CONCLUSION

Linear transformations in Minkowski space provide the mathematical foundation for describing changes between inertial reference frames in special relativity. Lorentz transformations preserve the Minkowski metric and therefore maintain the invariant spacetime interval. Their matrix structure demonstrates the fundamental unity of space and time and provides a geometric interpretation of relativistic phenomena.

Therefore, the study of linear transformations in Minkowski space establishes a direct connection between linear algebra, differential geometry, and the physical principles of special relativity.

REFERENCES

1. Einstein A. Relativity: The Special and General Theory. New York: Henry Holt and Company, 1920.
2. Taylor E. F., Wheeler J. A. Spacetime Physics: Introduction to Special Relativity. 2nd ed. W. H. Freeman, 1992.
3. Schutz B. A First Course in General Relativity. 2nd ed. Cambridge University Press, 2009.
4. Rindler W. Introduction to Special Relativity. 2nd ed. Oxford University Press, 1991.