MATHEMATICAL MODELING OF TURBULENT WATER FLOW IN AN ARCHIMEDES SCREW

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The Archimedes screw is one of the most promising hydromechanical devices for lifting water in open channels and efficiently utilizing the potential energy of flowing water. The performance of this device largely depends on the appropriate selection of its geometric parameters, including the screw diameter, helical pitch, inclination angle, screw length, and rotational speed. The rational determination of these parameters improves water-lifting efficiency, reduces energy consumption, and enhances the operational reliability of the system. Therefore, the mathematical modeling of the Archimedes screw and the determination of its optimal geometric parameters have become important research directions in the fields of hydromechanics and small-scale hydropower.

Mathematical modeling makes it possible to conduct a comprehensive analysis of the hydrodynamic processes occurring within the Archimedes screw, investigate the interaction between the water flow and the rotating screw, and predict the hydraulic performance of the device under various operating conditions. This approach significantly reduces the need for expensive and time-consuming experimental investigations, facilitates the identification of optimal design parameters, and improves the engineering design process. Consequently, it provides a scientific basis for the efficient utilization of flowing water energy in open channels, its conversion into mechanical energy, and the implementation of energy-efficient technologies. Furthermore, the Archimedes screw can be employed not only as a water-lifting device but also as a hydro turbine operating under low-head flow conditions, making it an important technology for the development of small-scale hydropower systems.

The flow around an Archimedes screw generally occurs at high Reynolds numbers and exhibits a turbulent nature. Therefore, reliable evaluation of the hydraulic performance of the device requires mathematical modeling of turbulent flow. In modern Computational Fluid Dynamics (CFD), the principal approaches for turbulence modeling include Direct Numerical Simulation (DNS), Large Eddy Simulation (LES), and the Reynolds-Averaged Navier–Stokes (RANS) method. The DNS approach provides the highest level of accuracy because it directly resolves all turbulence scales; however, it requires extremely high computational resources. In the LES approach, the large-scale turbulent eddies are resolved explicitly, whereas the smaller eddies are represented by subgrid-scale models. Although LES requires less computational effort than DNS, its computational cost remains considerable for complex three-dimensional flow problems. Consequently, the RANS approach is widely employed in engineering applications, including the analysis of water flow in Archimedes screws, because it provides an appropriate balance between computational efficiency and prediction accuracy.

The Reynolds number is the primary dimensionless parameter used to characterize the flow regime, representing the ratio between inertial and viscous forces. At low Reynolds numbers, viscous forces dominate, resulting in laminar flow. As the Reynolds number increases, inertial forces become dominant, causing flow instability and transition to turbulence. Turbulent flow is characterized by the complex interaction of vortices of different sizes. The kinetic energy generated within large eddies is progressively transferred to smaller eddies through the turbulent energy cascade until it is ultimately dissipated into thermal energy by molecular viscosity at the smallest scales. This mechanism is explained by Kolmogorov's theory of the turbulent energy cascade and constitutes one of the fundamental principles of modern turbulence theory.

Fluid motion is governed by the Navier–Stokes equations, which are derived from the conservation laws of mass and momentum. However, the direct numerical solution of these equations at high Reynolds numbers requires prohibitively large computational resources. Therefore, the time-

averaging technique proposed by Reynolds is commonly applied. In this approach, the velocity and pressure fields are decomposed into mean and fluctuating components, leading to the Reynolds-Averaged Navier–Stokes equations. As a consequence of the averaging process, additional unknown quantities known as the Reynolds stresses appear in the governing equations. To close the system of equations, the Boussinesq hypothesis is employed, whereby the Reynolds stresses are expressed through the concept of turbulent viscosity. This closure enables the governing equations to be solved using numerical methods.

In the present study, the Reynolds-Averaged Navier–Stokes (RANS) equations were employed to model the water flow around the Archimedes screw. This approach provides sufficient accuracy for describing the principal characteristics of turbulent flow in open channels, determining the velocity and pressure distributions, and evaluating the key factors affecting the hydraulic efficiency of the screw. The developed mathematical model will serve as a theoretical foundation for future studies aimed at optimizing the geometric parameters of the Archimedes screw, reducing hydraulic losses, and improving the overall hydraulic efficiency of the device.

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